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# Evaluating the Basle Guidelines for Backtesting Banks' Internal Risk Management Models

The 1996 Amendment to the Basle capital accord to incorporate market risks constitutes a breakthrough in the determination of capital requirements. Rather than dictating these requirements through a uniform supervisory approach, banks are allowed to use their own, internal models for computing the capital required. In order to mitigate moral hazard problems and stimulate banks to use adequate internal models, the models must be subjected to a backtesting procedure. If a model produces too many incorrect predictions, increased capital requirements result. This paper provides an evaluation of the current internal models approach in conjunction with the proposed backtesting procedure. In particular, using a stylized representation of the present supervisory framework, we investigate whether banks are provided with the right incentives to come up with the correct internal model. We find that, under the current regulatory framework, banks are prone to underreporting their true market risk. A much stricter penalty scheme is required in order to align banks' incentives with those of the supervisor. We check the sensitivity of our results to changes in the length of the planning horizon, portfolio risk, time preferences, risk attitudes, and the distribution of financial returns.

CAPITAL REQUIREMENTS play a major role in the banking industry [see, for example, Berger, Herring, and Szegö (1995) for an overview]. Following Estrella (1995), we can distinguish between market capital requirements and regulatory capital requirements. Market capital requirements serve to reduce agency problems, balance the benefits of tax evasion versus increases in financial distress costs, and reduce transaction costs if new investment capital is needed. Regulatory capital requirements, by contrast, aim at protecting the government (and ultimately the taxpayers) against financial distress costs and guaranteeing the soundness and stability of the financial system. From this regulatory or supervisory perspective,

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capital serves as a cushion to absorb part of the effect of adverse economic developments on financial institutions.

A key statistic in formulating capital requirements is the captial ratio, that is, the ratio of capital to total assets. A typical measure of capital is equity, possibly increased by the level of subordinated debt; see Berger, Herring, and Szegö (1995). They show that over the past 150 years U.S. capital ratios have gradually declined from around 40 percent in 1850 to about 6–8 percent from 1940 onward. Several sharp drops in capital ratios follow major changes in the regulatory framework. For example, the creation of the Federal Deposit Insurance Company (FDIC) in 1933 resulted in a decrease in capital ratios of about 50 percent over a ten-year period.

It is well known that flat premium deposit insurance schemes like that of the original FDIC can cause perverse incentives for banks, inducing them to engage in more risky activities (see, for example, John, John, and Senbet 1991). Boot and Thakor (1991) show that these effects may be mitigated by certain off-balance-sheet activities. The historically low capital ratios since the creation of the FDIC and the growing awareness of banks' misaligned incentives have spurred the introduction of new, risk-based capital requirements. The prime breakthrough is the Basle accord of 1988. Following this accord and its implementation in 1990, banks are required to hold a capital reserve between 0 percent and 8 percent for each transaction, depending on its riskiness, for example, government (0 percent) versus corporate (8 percent) loans. The accord also postulates capital requirements for certain off-balance-sheet activities. Avery and Berger (1991) and Berger, Herring, and Szegö (1995) argue that capital ratios have generally increased due to the new risk-based capital standards. Others comment that the requirement of 8 percent is ad hoc and insufficient to meet capital needs in times of stress (see Bradley, Wambeke, and Whidbee 1991).

Though the 1988 Basle accord already constitutes an important innovation to bank regulation by linking capital requirements to risk, still the accord has a limited scope. This is mainly due to the fact that the accord focusses on credit risk as the single most important risk factor in the banking sector. Due to the rapid developments in financial markets over the last decade, however, a second risk factor, namely market risk, has grown in importance considerably. Financial losses due to market movements can be large and occur very rapidly, especially if derivative instruments are involved. Some renowned examples include Orange County, Metallgesellschaft, and Barings. See also Jorion (1995). To accommodate the increased importance of market risk, supervisory standards had to be re-set. An important development in this respect is the Amendment to the 1988 Basle accord; see BCBS (1996b). During the final editing stage of this paper (early 2001), a complete revision of the 1988 accord has been proposed by the BIS. This revision encompasses the 1996 amendment for market risk, but also proposes new regulations for credit risk.

The Amendment has several innovative features; see also Hendricks and Hirtle (1997). First, capital requirements are based on banks' internal risk management models. Second, qualitative risk management standards are put to the fore. In particular the first of these two features constitutes a major departure from previous supervisory practices. Instead of dictating capital requirements through a uniform supervisory approach, banks are allowed to use their own purpose-tailored models and expertise for computing the capital required. This allows much more flexibility on the part of banks. The use of quantitative internal risk management models accompanied by a uniform supervisory standard for reporting the output of these models allows for a direct comparison of portfolio risk across time and institutions. This is one of the main advantages of the approach laid out in the Amendment (see Gizycki and Hereford 1998).

An obvious major drawback of the proposed internal models approach is the moral hazard problem induced by it. Supervisors have to make sure that banks use an adequate model as opposed to a model that produces capital requirements that are too low from a regulatory point of view (see also Danielsson, Hartmann, and de Vries 1997). In order to control this moral hazard problem, a model backtesting procedure has been put in place (see BCBS 1996a). The backtesting procedure automatically results in increased capital requirements if the internal model produces too many incorrect forecasts of market risk. For details on the backtesting procedure, I refer to BCBS (1996a) and section 1 below.

The aim of the present paper is twofold. First, we investigate whether the internal models approach combined with the current framework for model backtesting induce the right incentives for banks to come up with good internal models. In particular, we address the question whether banks are likely to underreport their market risk or not, for example, by employing a less conservative internal model. Using a stylized representation of the current regulatory framework, we find that banks are indeed prone to underreporting under a variety of conditions. Given this result, the second question we address in this paper concerns the construction of optimal backtesting procedures. In particular, we look for a backtesting framework with corresponding penalty scheme that provides banks with sufficiently strong incentives to construct adequate risk management models. It turns out that with respect to the present backtesting penalty scheme (BCBS 1996a), a much stricter scheme is called for in order to align banks' incentives with those of the supervisor.

The adequacy of the internal models approach as a useful tool for supervision has been debated earlier in the literature by proponents of the precommitment approach (see, for example, Kupiec and O'Brien 1995a,b, 1996, 1997). They have been contradicted by, for example, Gumerlock (1996), who argues that the internal models approach, when used in conjunction with a backtesting procedure, has several advantages over the precommitment approach. Kupiec (1995), however, argues that given the information regularly available to the supervisory institution, it is difficult to develop good statistical backtesting procedures that enable one to detect fraud models at an early stage. Although the aim of the present paper is not to resolve this controversy completely, our results can be used to assess the adequacy of the present Basle proposals in preventing excess risk-seeking behavior and systematic underreporting of the true market risk.

The paper is set up as follows. In section1, the basic framework is laid out. A stylized representation of the current regulatory framework is presented. Section 2 contains numerical results on the evaluation of the present Basle guidelines. Section 3 describes the design of optimal backtesting procedures from the supervisory point of view. Concluding remarks and suggestions for future research are contained in section 4.

## 1. A STYLIZED REPRESENTATION OF CURRENT REGULATIONS

As a starting point, we have to define the way risk is measured. Following BCBS (1996a,b), market risk is quantified using the concept of value at risk (VaR). VaR is the maximum loss that can occur during a certain period of time given a certain confidence level. In our setting, the VaR corresponds to a specific quantile of the profit/loss distribution of the bank's portfolio. For a textbook treatment on VaR, see Jorion (1997). The BCBS (1996b) guidelines specify a ten-day VaR and a 99 percent confidence level, that is, the maximum loss that can occur with a 99 percent probability in a ten-day period. The ten-day period is motivated by the fact that in periods of severe market stress, liquidity can evaporate very quickly. In such cases, it may be difficult to unwind unprofitable parts of the portfolio in timely fashion.

The bank's internal risk management model is used to compute the VaR. This VaR has to be reported to the supervisor on a daily basis (see BCBS 1996a), along with realized profit and loss figures. Let VaR($t$) denote the VaR reported to the supervisor at time $t$. Then the capital requirement is equal to a multiple $f(t)$ of the average VaR($t$) over the last sixty days. If the computed capital requirement is less than the previous day's VaR, the latter is used instead.[1] In order to keep our model analytically manageable, we have to make several abstractions. First, we focus on a fixed portfolio of assets and liabilities. We thus abstain from considering the interaction of supervisory regulations and active asset and liability management of banks. This is a limitation of the present framework and we come back to it in the concluding section of this paper. In any case, holding the portfolio composition fixed limits the bank's possibilities of exploiting the loopholes in the regulatory framework. Second, we abstract from the difference between VaR used for reporting and the VaR used for computing capital requirements. The former applies to a one-day period, while the latter relates to a ten-day period. The difference is immaterial for the derivations in the remainder of the paper, where we assume that financial day-to-day returns are independent and identically distributed (i.i.d.). In that case, given the fixed portfolio, the ten-day VaR is just a fixed multiple of the one-day VaR. Of course, the assumption of i.i.d. returns is not realistic, as a prominent feature in financial markets is time dependence, for example, volatility clustering (see Pagan 1996). We abstract from this complication, as it would render the model intractable. Time dependence is, however, a crucial component for a practical evaluation of VaR measures; see Christoffersen (1998) for some possible methods. See also Christoffersen and Diebold (1997), Christoffersen,

1. In the official guidelines, there is another capital charge to account for idiosyncratic risk. For simplicity, we abstract from that here. See also Lopez (1998, 1999).

Diebold, and Schuermann (1998), and Danielsson and de Vries (1997) for further critical remarks on the (un)importance of time dependence for VaR calculations.

The scalar inflation factor $f(t)$ used for transforming the VaR into a capital requirement is one of the crucial variables in this paper. It allows the supervisor to impose monetary penalties on banks with incorrect internal models by the use of a backtesting procedure. Typically, $f(t) = 3$, which does not appear too unreasonable from a prudential perspective if one allows for model misspecification (see Stahl 1997 and Hendricks and Hirtle 1997). Assessing the correctness of the VaR model is not an easy task. Most tests are hampered by a lack of statistical power (see Kupiec 1995). See also Lopez (1998,1999) and Berkowitz (1999) for several possible ways to evaluate VaR models, and Cassidy and Gizycki (1997), Pritsker (1997), Jackson, Maude, and Perraudin (1998), Engel and Gizycki (1999), and van den Goorbergh and Vlaar (1999) for simulation evidence on the performance of different VaR estimation and backtesting methods. In this paper we follow the backtesting methodology of BCBS (1996a). In particular, VaR figures and actual loss figures are reported on a daily basis to the supervisor over a 250-day period. Given a VaR confidence level of 99 percent, we expect an average number of 2.5 days for which the realized loss exceeds the reported VaR, that is, a so-called VaR violation. If the number of days is significantly larger, the internal model becomes suspect as it potentially generates too low VaR numbers and, therefore, too low capital requirements. In that case, the supervisor can decide to increase the scaling factor $f(t)$ or (in the extreme) to forbid the use of the internal model altogether. The Basle proposal for backtesting has a very simple mapping from the number of VaR violations to the scaling factor. See Table 1, which distinguishes three zones. In the green zone, the number of violations over the past 250 days is small, such that the internal model can be deemed adequate for capturing the true market risk. Consequently, no increase in the VaR

TABLE 1

VaR Multiplication Factors after Backtesting

| Zone | Number of exceptions ($i$) | Scaling factor ($\tilde{f}(i)$) |
|---|---|---|
| Green Zone | 0 | 3.00 |
| | 1 | 3.00 |
| | 2 | 3.00 |
| | 3 | 3.00 |
| | 4 | 3.00 |
| Yellow Zone | 5 | 3.40 |
| | 6 | 3.50 |
| | 7 | 3.65 |
| | 8 | 3.75 |
| | 9 | 3.85 |
| Red Zone | ≥10 | 4.00 |

Notes: This table contains the number of violations during a 250-day period of reported 1 percent VaR figures by realized losses on a bank's portfolio and the corresponding BCBS scaling factor $\tilde{f}(\cdot)$ for VaR for determining the capital reserves associated with market risk.
Source: Basle Committee on Banking Supervision (1996a), Table 2.

scaling factor is required from its initial value $f(0) = 3$. In the yellow zone, doubt arises as to the integrity and/or validity of the bank's model. This is reflected in the gradual increase in the VaR scaling factor. If the number of violations is larger than or equal to 10, that is, if the reported VaR is a 4 percent VaR rather than a 1 percent VaR, the model is judged inadequate. In that case, the scaling factor is raised to 4 and the bank is likely to be obliged to revise its internal risk management model.

For the regulatory evaluation period we select one year. This means that the factor $f(t)$ is set once a year and is in place for the complete subsequent year. In practice, the supervisor will be able to react more promptly to excessively high numbers of VaR violations, for example, every quarter. By using a more frequent updating scheme for $f(t)$, however, tractability is lost. In particular, if the regulatory frequency is higher than 250 days while the backtesting still takes place over a 250-day period, subsequent observations on the number of VaR violations are correlated. As these observations are put through the nonlinear mapping $\tilde{f}(\cdot)$ in Table 1, the analytics become extremely complicated. To check the sensitivity of our results with respect to the regulatory frequency, we have performed an experiment in which the factor $f(t)$ was set (and left in place) every 63 days. To avoid the correlation mentioned earlier and regain tractability, the number of violations was also counted for 63 instead of 250 days. The number of violations was multiplied by 4 to make the mapping in Table 1 applicable. Using this quarterly approach instead of the annual approach adopted in the remainder of this paper did not have a substantial impact on the conclusion. Relative to the annual regulatory frequency, using quarterly updates resulted in both lower and higher degrees of underreporting depending on the particular model parameters, see also further below.

We now turn to the behavior of the bank. First note that capital requirements have a direct impact on the bank's profit opportunity set. Therefore, abstracting from its own incentive to hold capital reserves, see Estrella (1995), the capital reserves required by the supervisor are undesirable for the bank from a pure profit point of view. As a consequence, it has an impetus to report VaR figures that are too low, as this lowers its costs of capital. Low VaR figures, however, result in an increased probability of VaR violations, and, therefore, in potential increases in future costs of capital through increases in $f(t)$. This provides a disincentive for the bank to underreport its true market risk. The bulk of this paper now concerns which of these two effects prevails under various conditions. To simplify matters considerably, we consider only a once-and-for-all choice by the bank of its internal model. This is in line with the simplifying assumption of a fixed portfolio composition. Note that in the present framework choosing an internal model is tantamount to selecting a particular VaR level. By allowing this VaR level to be chosen only once, we restrict the bank's flexibility in exploiting the weaknesses of the backtesting procedure. So if this simplification causes a bias in our results, the bias is more likely to work against than in favor of finding underreporting of market risk.

To complete the description of the regulatory framework, we have to formalize what happens when a bank enters the red zone from Table 1. Let VaR* denote the true VaR level of the bank's portfolio. Note that VaR* does not depend on time given the fixed portfolio composition. We can define a constant $c$ such that the VaR level chosen for regulatory reporting purposes ($VaR_r$) equals

$$VaR_r = (1 - c) \cdot VaR^* . \tag{1}$$

The constant $100 \cdot c$ can be interpreted as the percentage of underreporting with respect to the true VaR. We now assume that the bank can report $VaR_r$ to the supervisor up to the moment when the red zone is entered. After that, the regulator sets $c = c^*$ for all remaining periods. In the remainder of this paper we use $c^* = 0$, such that the VaR is set to the true VaR once the red zone is entered. The supervisor can try to achieve this by close inspection of the bank's internal model. It may seem somewhat optimistic to assume that the supervisor will always be able to set $c^* = 0$. Describing the risk of a bank's portfolio requires expert insight into the operation and interaction of all the bank's financial instruments. Such expert knowledge may not always be at hand within the supervisory institution, for example, due to time constraints. Therefore, it may well be the case that $c^*$ is either positive or negative. We comment on the sensitivity of our numerical results with respect to the choice of $c^*$ in section 2. If $VaR(t)$ is the VaR level reported at time $t$, we now have

$$VaR(t) = \begin{cases} VaR_r & \text{if } f(s) < 4 \text{ for all } s \leq t , \\ VaR^* & \text{otherwise .} \end{cases} \tag{2}$$

Similarly, we define

$$c(t) = \begin{cases} c & \text{if } f(s) < 4 \text{ for all } s \leq t , \\ c^* & \text{otherwise .} \end{cases} \tag{3}$$

Given our stylized representation of the regulatory framework, we assume the bank minimizes a power function of all present and future expected opportunity costs that are attributable to the market risk capital requirements, that is,

$$\min_c \sum_{t=0}^{T-1} E_0\{e^{-\rho t} \cdot U[r \cdot VaR(t) \cdot f(t)]\} , \tag{4}$$

where $U(x) = x^\gamma$, with $\gamma \geq 1$. The operator $E_0(\cdot)$ denotes the expectations operator given the available information at time 0. Furthermore, $r$ is an opportunity cost rate, $\rho$ is the time preference parameter, and $T$ is the planning horizon. The minimization in (4) is carried out with respect to the level of underreporting $c$. The opportunity cost rate $r$ is either a required internal rate of return or a market rate. Note that for $\gamma = 1$ (4) collapses to the minimization of expected opportunity costs. For $\gamma > 1$, larger opportunity costs are weighted more heavily, such that stable costs are preferred to highly variable costs, cf. power utility functions for consumption. By varying $\gamma$ we can check whether the risk attitude of the bank itself has an effect on the extent of underreporting.

The stochastic variables in (4) are $f(t)$, $t = 1,...T - 1$. Given the postulated VaR evaluation period of 250 trading days combined with the assumption of i.i.d. daily returns, the $f(t)$s are independent and follow a binomial distribution with parameters $n = 250$ and

$$p = p(c(t)) = P[\Pi(t) < -VaR(t)] = P[\Pi(t) < -(1 - c(t)) \cdot VaR^*] \,, \qquad (5)$$

where $\Pi(t)$ denotes the bank's profit. See also BCBS (1996a). Note that the probability of success $p$ changes once the bank enters the red zone, that is, once $VaR(t)$ changes from $VaR_r$ to $VaR^*$. Using (4) and (5), it is easy to see that lowering the reported VaR has two effects. First, there is a direct effect on the objective function (4), because the reserve requirement based on the reported VaR causes opportunity costs to the bank. Second, lowering the reported VaR increases the probability $p$ in (5), such that the expectation of $f(\cdot)$ is increased. This results in a larger penalty for future VaR figures in the objective function.

Although the present framework allows us to address several important questions related to supervision and the internal models approach advocated by the BCBS, there are also several limitations. First, as mentioned before, we do not consider the interaction between supervisory regulations and active asset and liability management by banks. Active balance sheet management can be used as an additional instrument by the bank for reducing the number of future VaR violations if an increase in the VaR scaling factor becomes more likely due to the number of VaR violations already realized in the course of the year. Second, we have not modeled any credibility issues related to a large number of VaR violations. Credibility issues could play a role in the relationship between the bank and the supervisor or between the bank and its customers. Third, the present framework does not incorporate the guidelines of the BCBS on auxiliary model testing, such as stress testing, see the BCBS (1996b). Stress tests reveal the internal model's behavior under extremely adverse market circumstances and can in certain cases trigger a prompter reaction from the side of the supervisor. Finally, we have taken a simplistic pure profit point of view for the bank. Of course, apart from a profit motive every bank has a drive to manage its returns as well as its risks. This means that the bank has an impetus on its own to hold capital reserves if its market risk is high. This can be captured by imposing upper bounds on the allowable values of $c$ in (4). Estrella (1995) even argues that the regulatory capital requirement may lie below the market capital requirement, as discussed in the introduction. Strong empirical support for this claim, however, is difficult to obtain. Therefore, it is useful to know whether or not the bank's incentives are in line with those of the supervisor under the current regulations if the market capital requirement is not binding. Also note that up to a certain extent we can check the sensitivity of our results with respect to the risk attitude of the bank by varying $\gamma$ in (4).

## 2. EVALUATION OF THE PRESENT GUIDELINES

In this section we conduct some numerical experiments in order to assess the effectiveness of the present BCBS (1996a) proposals for backtesting internal risk management models. For ease of notation, we set $r \cdot VaR^* = 1$. This does not effect the results on underreporting.

In the Appendix we prove that the objective (4) can be rewritten as

$$\min_c \left\{ U[f(0)\cdot(1-c)] + \frac{1-(\xi_1 e^{-\rho})^{T-1}}{1-\xi_1 e^{-\rho}} e^{-\rho}(U_0 - U^*) + \frac{1-e^{-\rho(T-1)}}{1-e^{-\rho}} e^{-\rho} U^* \right\}, \quad (6)$$

where $\xi_1$, $U_0$, and $U^*$ are defined in (A4), (A5), and (A6) in the Appendix, respectively. In fact, $\xi_1$ is the probability of entering the red zone, given that the red zone has not been entered before. Furthermore, $U_0$ and $U^*$ denote the expected cost penalty function given that the red zone has not and has been entered before, respectively. In our experiments, we use $e^{\rho} = 1.1$, such that the discount rate is 10 percent. The sensitivity to the value of $\rho$ is discussed further below.

Equation (6) has a clear economic interpretation. The first term is the disutility of the initial period's opportunity cost. The second term gives the probability weighted discounted sum of expected losses due to the use of the self chosen level of underreporting $c$ rather than the regulatory standard $c^*$. The last term is the expected discounted sum of disutilities following from the benchmark regulatory model $c^*$, that is, the true model. Note that this last term does not depend on $c$, such that the solution to (6) coincides with that of

$$\min_c \left\{ U[f(0)\cdot(1-c)] + \frac{1-(\xi_1 e^{-\rho})^{T-1}}{1-\xi_1 e^{-\rho}} e^{-\rho}(U_0 - U^*) \right\}. \quad (7)$$

Increasing the level of underreporting has several effects. First, the initial opportunity costs are decreased. Second, the probability of entering the red zone, that is, $\xi_1$, is increased. Third, the disutility gain $U_0 - U^*$ is affected in three ways: (i) the probability of VaR violations increases, (ii) the expected penalty factor $f(t)$ increases, and (iii) the opportunity costs given that the red zone is not entered, decrease. The direction of the composite effect is not clear a priori.

Before we can actually compute the optimal value of $c$, we have to specify a functional form for the probability measure $P[\cdot]$ in (5). We assume that the profit $\Pi(t)$ follows a Student $t$ distribution with $\nu$ degrees of freedom. The Student distribution nests the familiar normal distribution for $\nu \to \infty$. By considering Student $t$ distributions instead of the normal, we can study the effect of leptokurtosis on the optimal model choice of the bank. Leptokurtosis is a common phenomenon in financial markets (see, for example, Pagan 1996 and Campbell, Lo, and MacKinlay 1997). Define

$\mu_\Pi$ and $\sigma_\Pi$ to be the mean and standard deviation of $\Pi$, respectively. Furthermore, let $T_\nu(\cdot)$ be the cumulative distribution function (c.d.f.) of the standard Student $t$ distribution with $\nu$ degrees of freedom, and let $T_\nu^{-1}(\cdot)$ denote the inverse standard c.d.f. Finally, let $SR$ be the Sharpe ratio of the profit random variable, that is, $SR = \mu_\Pi/\sigma_\Pi$. Using these definitions and the fact that for a 99 percent confidence level VaR,

$$VaR^* = -\mu_\Pi - (1 - 2\nu^{-1}) \cdot \sigma_\Pi T_\nu^{-1}(0.01) ,$$

$p(c)$ from (5) can be rewritten as

$$p(c) = T_\nu\left((1-c)\cdot T_\nu^{-1}(0.01) - \frac{c \cdot \nu \cdot SR}{\nu - 2}\right). \tag{8}$$

It follows directly from (8) that $p(0) = 1\%$, such that the use of the true model leads to the correct coverage level of 99 percent. Furthermore, (8) reveals that the probability of future VaR violations not only depends on the model chosen, that is, on $c$, but also on the degree of fat-tailedness ($\nu$) of profits and on the overall risk of the bank's portfolio through the presence of the Sharpe ratio $SR$. Banks with high Sharpe ratios profit less from choosing safe models ($c < 0$) in terms of reductions in $p(c)$ than banks with low Sharpe ratios. This is most easily understood by considering two banks with the same value of $\sigma_\Pi$, but different Sharpe ratios. In that case, the bank with the higher Sharpe ratio has a safer portfolio and, thus, a smaller true VaR figure. Increasing the true VaR by the same percentage amount for both banks, therefore, reduces the probability of VaR violations comparatively more for the bank with the low Sharpe ratio, because the nominal shift in the VaR for this bank is larger than for the bank with the high Sharpe ratio.

Figure 1 presents graphs of the objective function for the normal distribution ($\nu \to \infty$). The objective function is plotted for two different values of the planning horizon $T$, several values of the Sharpe ratio $SR$, and for a linear ($\gamma = 1$) and a quadratic ($\gamma = 2$) cost penalty function.

Several things can be noted in Figure 1. First of all, a local minimum of the objective function is situated at $c > 0$ for all curves presented. This means that banks have an incentive to choose overly risky internal risk management models. We concentrate on the local minimum for the remaining discussion and call it the *optimal* value of $c$. This is not unreasonable, given there is a natural upper bound on the values of $c$ allowed by the supervisor. According to the BCBS (1996b) guidelines, the reported VaR based on the internal model may not fall below 50 percent of the VaR based on the standard BCBS approach. This puts an upper bound on the allowable values of $c$. For empirical evidence on the relation between the standard approach and the internal models approach, see Gizycki and Hereford (1998). Furthermore, the bank possibly has its own upper bound for allowable values of $c$ due to risk aversion motives which are not incorporated in the present framework [see the discussion in section 1 and Estrella (1995)]. The magnitude of the optimal $c$ is substantial, indicating that it

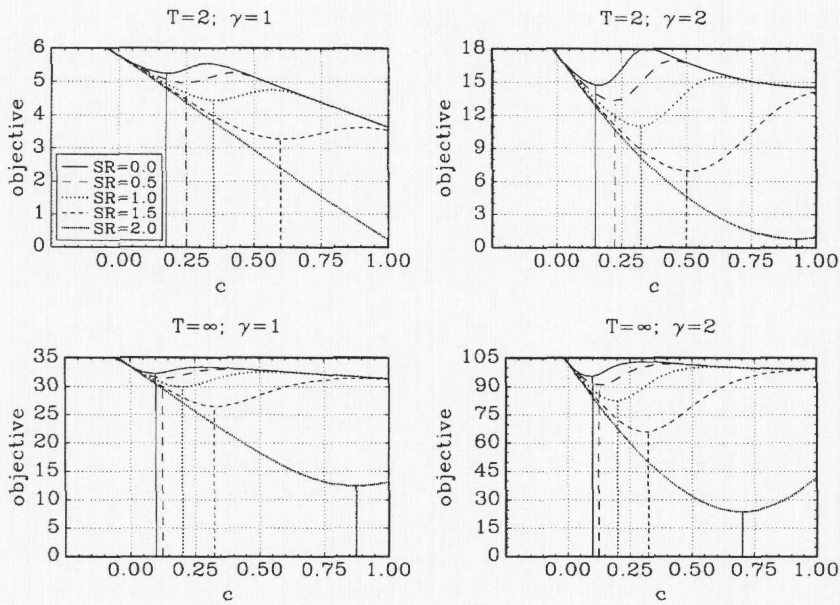


FIG. 1. The Objective Function (6) for Different Values of the Sharpe Ratio ($SR$), Different Lengths of the Planning Horizon ($T$), and Different Values for the Power Exponent of the Cost Disutility Function ($\gamma$). $100 \cdot c$ denotes the percentage of underreporting, that is, the decrease of the reported VaR ($VaR_r$) with respect to the true VaR ($VaR^*$). At the local minimum of each curve, a vertical line indicates the $c$-value corresponding to this local minimum. The discount rate used for constructing the plots is 10 percent. The bank has to adopt the true model if it enters the red zone of Table 1, that is, $c^* = 0$. The plots are made for $r \cdot VaR^* = 1$.

is optimal for the bank to underreport its true market risk by 25 percent or more if $T = 2$ and $SR > 0.5$. For longer planning horizons, the mismatch between true and reported VaR is generally smaller, but still substantial. The decrease in the optimal value of $c$ for larger values of the planning horizon $T$ is intuitively clear. If more future opportunity costs are taken into account and if the internal risk model must be chosen once and for all, then a safer risk management model will, ceterus paribus, result in a smaller value of the objective function. The effect is more pronounced if we set $c^* < 0$ (not shown). In that case the expected future opportunity costs of the bank increase because the default model ($c^*$) sets a higher VaR for reporting purposes than the true VaR. Even in this case, however, the optimal values of $c$ remain negative, such that it is still profitable for the bank to report underestimates of its true VaR to the supervisor. Extreme parameter configurations are needed (for example, $T = \infty$ and $c^* = 2$ for $\gamma = 2$, and $T = \infty$ and $c^* = 3$ for $\gamma = 1$) to drive the optimal value of $c$ to 0.

Second, and related to the first characteristic of Figure 1, the optimal degree of underreporting $c$ decreases if the discount rate $\rho$ is smaller (not shown). Smaller discount rates imply that future opportunity costs are weighed more heavily in the objective function. Choosing a reporting VaR level below the true market risk, that is, $c > 0$, causes an increase in (expected) opportunity costs during future periods. Consequently, the optimal value of $VaR_r$ (or $-c$) is decreasing in the discount rate $\rho$. Note that for extreme discounting $\rho \to \infty$, only the first period opportunity costs are taken into account, such that the objective function becomes monotonic in $c$ for $\gamma = 1$.

Third, note the nonmonotonic shape of the objective functions in $c$. We focus on the linear case $\gamma = 1$. The quadratic case $\gamma = 2$ reveals similar features. For $c = 1$, the reported VaR in the first period equals 0. Of course, this is an unreasonable value given the BCBS lower bound on reported market risk levels mentioned above. By looking at $c = 1$, however, it is clear that for significant underestimates of the true VaR, the bank is (almost surely) forced in the next period to adopt the true model as an internal risk management model, that is, to set $c(t) = c^* = 0$ for $t \geq 1$. Consequently, for extreme under-estimates of the true VaR, the objective function becomes linear in $c$ with slope coefficient $-f(0) = -3$. For extreme overestimates of the true VaR, the opposite happens. In that case future VaR violations become extremely unlikely, such that the objective function in (7) effectively collapses to

$$(1-c)\cdot f(0) + \frac{1-e^{-\rho(T-1)}}{1-e^{-\rho}} e^{-\rho}(c^* - c)\cdot f(0), \tag{9}$$

which is again linear in $c$ with a slope coefficient greater than $-3$. Between these two extremes, there is a range of values for $c$ to link the two different linear segments. It is in this range that the trade-off between reductions in present opportunity costs versus an increase in expected future opportunity costs becomes really apparent through the nonmonotonic behavior of the objective function.

Fourth, the Sharpe ratio has an increasing effect on the optimal value of $c$ in the figures presented. Banks with less-risky portfolios in terms of higher Sharpe ratios choose a higher percentage of underreporting. As mentioned earlier, a bank with a high Sharpe ratio profits less in terms of a percentage reduction in expected future opportunity costs when the bank increases its reporting VaR. Therefore, the bank with a large Sharpe ratio places more emphasis on reducing present instead of future opportunity costs. This results in higher values of $c$ chosen by banks with higher Sharpe ratios.

Fifth, $\gamma$ has a decreasing effect on the optimal degree of underreporting. This is clear, as a higher penalty on increases in opportunity costs induces a shift towards a more truthful reporting of market risk. Another way of looking at this is from a risk aversion perspective. For $\gamma = 2$, the bank prefers stable costs to variable ones. Higher $c$ values lead to a higher probability of facing future increases in $f(\cdot)$ and entering the red zone, and thus to unstable opportunity costs. Therefore, lower values of $c$ are preferred for $\gamma = 2$ versus $\gamma = 1$.

We now turn to a discussion of the robustness of our results with respect to the degree of fat-tailedness of the profit distribution. The degree of fat-tailedness can be tuned by setting the parameter $\nu$. The larger the value of $\nu$, the more the profit distribution resembles the familiar normal distribution. Changing $\nu$ triggers several different effects; see equation (8). First, decreasing $\nu$ shifts the 1 percent quantile $T_\nu^{-1}(0.01)$ to the left. Second, for lower values of $\nu$ the Sharpe ratio becomes more important for the effect of $c$ on the probability of VaR violation $p(c)$. If two banks have the same Sharpe ratio, the bank with the fatter-tailed profit distribution profits less from a decrease in the degree of underreporting $c$. Third, there is an effect of $\nu$ through the c.d.f. $T_\nu(\cdot)$ used to compute $p(c)$ in (8). The total impact of the combination of these three effects is difficult to predict a priori. Therefore, we compute the optimal value of $c$ using the objective function (6) different values of the planning horizon $T$ and different degrees of leptokurtosis $\nu$. We focus exclusively on the linear case, $\gamma = 1$. The results are presented in Figure 2. Note that we use $3/\nu$ instead of $\nu$ as a plotting variable for reasons of layout. The normal distribution now corresponds to $3/\nu = 0$, while the most fat-tailed (finite variance) distribution considered is $3/\nu = 1 \Leftrightarrow \nu = 3$.

Note that in order to construct Figure 2, the Sharpe ratio is kept fixed. In effect, this means that the variance of the profit distribution is kept fixed as the degree of fat-tailedness $3/\nu$ is increased. This leads to a composite effect. First, larger values of $3/\nu$ lead to fatter tails, such that extreme profits become more likely. Second, if the variance is held fixed, larger values of $3/\nu$ lead to an increased precision $(\nu/(\sigma_\Pi \cdot (\nu - 2)))$ of the Student $t$ distribution and, thus, to a decrease in the probability of extreme profits. For more details on this, see Lucas and Klaassen (1998).

The decrease in the optimal value of $c$ as a function of $T$ for fixed $\nu$ is evident in Figure 2. Furthermore, the optimal value of $c$ is increasing in the degree of leptokurtosis $3/\nu$ for fixed $T$. This means that the effect of $\nu$ through a reduced effect of $c$ on

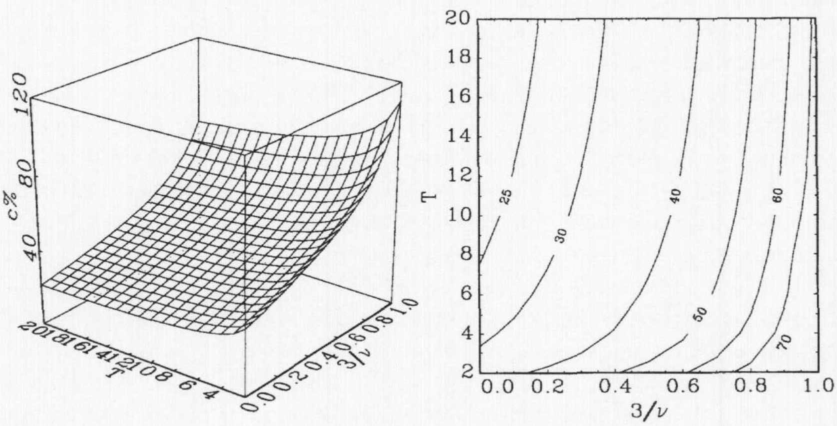


FIG 2. The (Locally) Optimal Degree of Underreporting, that is, the Percentage Decrease of the Reported VaR with respect to the True VaR ($100 \cdot c$). The objective function used is (6) with $\gamma = 1$. The Sharpe ratio used for constructing the figure is $SR = 1.0$. The optimal value of $c$ is graphed as a function of the planning horizon $T$ and the degree of fat-tailedness $3/\nu$. Higher values of $3/\nu$ indicate that the underlying distribution is more fat-tailed. Discounting takes place at a 10 percent rate, while the default risk management model is the true model, that is, $c^* = 0$.

$p(c)$ governs the composite effect mentioned above. Banks with a higher value of $3/\nu$ obtain relatively less reward from raising their reported VaR above the true level ($c < 0$) in terms of a decrease in expected future opportunity costs; see (8). Consequently, for short planning horizons, these banks will be more inclined to emphasize reductions in present opportunity costs by choosing a, ceterus paribus, higher value of $c$. Figure 2 reveals that this effect may be so strong that the bank will even opt for reporting VaR values more than 70 percent below the true risk level if the profit distribution is fat-tailed, for example, $3/\nu = 1$. Note again that such figures may not be realized in practice due to banks' market capital requirements (Estrella 1995), and the BCBS safety check using the standard rather than the internal model, see our earlier discussion.

To conclude this section, we summarize the main findings. If a bank is forced by the supervisory institution to pick an internal risk management model for the entire planning period, and if this model is subjected to backtesting according to the BCBS (1996a) report, banks generally select overly risky models. The effect is more pronounced for shorter planning horizons, higher discount rates $\rho$, fatter tails for the profit distribution, higher values of the Sharpe ratio, and linear versus quadratic cost penalties. By contrast, if the default model upon entering the red zone overestimates the true VaR, that is, if $c^* < 0$, relatively more prudent risk management models are chosen. Extreme parameter configurations are needed to drive the (locally) optimal value of $c$ to 0.

## 3. DESIGNING OPTIMAL BACKTESTING PROCEDURES

So far, we have concentrated on the optimal choice of the bank's reporting VaR given the supervisory regulations as laid out in BCBS (1996a, b). We now turn to a second important question. Given the bank's incentive to underreport the true market risk, what is the optimal backtesting approach for the supervisor? It is clear from the previous section that the monetary penalties as proposed by BCBS (1996a) are insufficient to guarantee a close match between reported and true VaR. We expect, therefore, that optimal backtesting procedures will set much higher monetary penalties than the ones presented in section 1. It is the aim of the present section to quantify such optimal penalty schemes and associated backtesting procedures.

Before we can proceed with the analysis, some choices must be made regarding the objectives of the supervisor and the instruments available for achieving these objectives. Note that the bank's optimal value of $c$ depends on several parameters, namely the planning horizon $T$, the discount rate $\rho$, the bank's Sharpe ratio $SR$, the power of the disutility function $\gamma$, and the degree of fat-tailedness of the profit distribution as characterized by the degrees of freedom parameter $\nu$. We assume that the bank has some prior ideas concerning the values of the above parameters. These ideas, possibly updated by empirical research, are summarized in the form of a (posterior) distribution function $q(T,\rho,SR,\nu)$. The supervisor now minimizes the objective function

$$E_q((c)^2 \cdot 1_{\{c \geq 0\}} + (c/\alpha_1)^2 \cdot 1_{\{c<0\}}) \,, \qquad (10)$$

where $E_q(\cdot)$ denotes the expectations operator with respect to the posterior distribution $q(\cdot)$, and $1_A$ is the indicator function of the set $A$. Equation (10) states that the supervisor minimizes an asymmetric quadratic loss function of the percentage mismatch between the true VaR and the reported VaR. If this minimization has to be carried out for known values of $T$, $\rho$, $SR$, and $\nu$, the posterior distribution can be chosen to have a unit mass point at the known parameter values. Alternatively, if an adequate backtesting procedure is needed for a broader range of parameter values, a nondegenerate support of the posterior distribution can be chosen. Note that $\alpha_1$ in (10) determines the degree of asymmetry. For $\alpha_1 = 1$, underreporting and overreporting are penalized in the same way. By contrast, if $\alpha_1 > 1$, underreporting is penalized relatively more heavily. An asymmetric loss function seems adequate from a supervisory point of view, where underreporting is of greater concern than overreporting. Note, however, that the supervisor may also have an incentive not to distort the market too much by excessively stringent regulations, that is, by inducing $c < 0$. This can be taken into account by setting $\alpha_1 < \infty$.

In this section we treat $\rho$ and $T$ as given. Moreover, we only consider the linear cost function, $\gamma = 1$. The VaR mismatch is thus averaged with respect to $SR$ and $\nu$ only. The results are remarkably stable with respect to variations in $\rho$ and $T$, such that we only report the findings for $e^{\rho} - 1 = 10\%$ and $T = 10$. We assume that the remaining posterior distribution $q(SR,\nu)$ is uniform on a grid of values for $(SR,\nu)$. We consider $\nu = 5, 10, \infty$ and $SR = \nu \cdot SR'/(\nu - 2)$, with $SR' = 0.5, 1.0, 1.5$. The limited number of combinations taken into account is motivated by two reasons. First, the optimization problem using objective function (10) is very computer intensive. A simple choice for the posterior distribution can speed up the calculations considerably. Second, the optimal solution is mainly driven by the corners of the $(SR,\nu)$-grid considered; cf. Figure 2. Therefore, we do not expect to lose much information when discarding many intermediate combinations of Sharpe ratios and degrees of freedom parameters.

We now turn to the available instruments for minimizing the objective function (10). Given the framework laid out in sections 1 and 2, we have as possible instruments the length of the backtesting period $n$, the penalty function $\tilde{f}(i)$, and the choice of the default model $c^*$. We concentrate on the first two of these. The value of $c^*$ is zero in all computations presented below. Negative values of $c^*$ result in a permanent increase in opportunity costs. Using the present value of these opportunity costs, negative values of $c^*$ can be represented by high temporary penalties, that is, by high values of $\tilde{f}(i)$. Some unreported experiments reveal that the qualitative results of the present section remain unaltered if the annual backtesting procedure ($n = 250$) is shortened to a biannual or a quarterly frequency, see also the comments in section 1. Therefore, we only present the results for $n = 250$. Note, however, that shortening the backtesting period would have adverse effects on the general statistical reliability of the backtesting procedure; see Kupiec (1995).

We estimate the function $\tilde{f}(i)$ nonparametrically. In particular, we set $\tilde{f}(0) = 3$ and

$\tilde{f}(250) = \alpha^*$, where $\alpha^*$ denotes an upper bound which is imposed to obtain numerical stability. Moreover, we restrict $\tilde{f}(\cdot)$ to be nondecreasing. The minimization problem now goes as follows. For a given penalty function $\tilde{f}(\cdot)$ satisfying the restrictions above, the banks set their (locally) optimal degree of underreporting $c$. The $c$ values are put into the loss function inside the expectations operator in (10). The loss function values and subsequently averaged overall values in the grid for ($SR$,$\nu$) are considered. This yields the objective function value (10) of the supervisor. The supervisor now minimizes this objective function with respect to $\tilde{f}(\cdot)$. The results are presented in Figure 3.

It is immediately apparent from the left-hand panel in Figure 3 that the optimal form of $\tilde{f}(i)$ does not look at all like the penalty function proposed by the BCBS (1996a). The corresponding figures for $\alpha_1 = 1,2$ look extremely similar, except that the values of $\tilde{f}(5)$ and $\tilde{f}(6)$ are slightly lower for lower values of $\alpha_1$ for $\alpha^*$ equal to 5 and 10, respectively. Three main differences can be seen with respect to the Basle function. First, the number of violations for which no additional penalty is imposed is larger for the optimal penalty function if the upper bound $\alpha^*$ is either sufficiently large or sufficiently small. Only for $\alpha^* = 5$ the number of no-penalty values of $i$ coincides with the Basle scheme. Second, while the penalty function proposed by the BCBS shows a gradual increase from the base factor 3 to the maximum factor 4, the optimal $\tilde{f}(i)$ is a kind of step function. Up to a certain threshold for VaR violations, no penalties are imposed on the bank. If the number of VaR violations exceeds this threshold, however, a comparatively constant penalty is imposed. For large enough

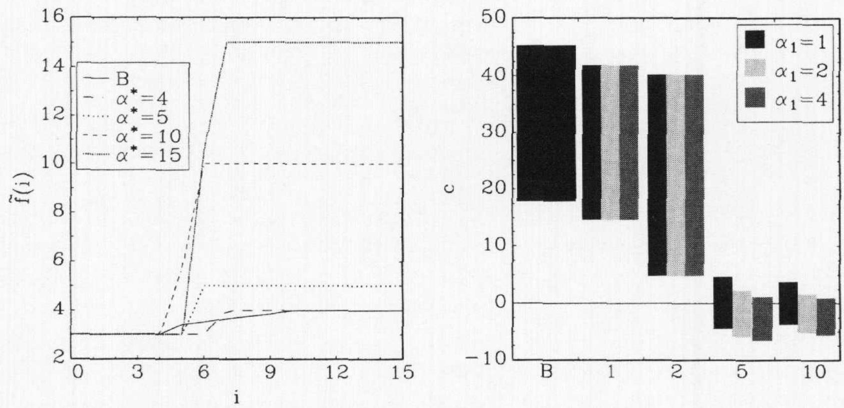


FIG. 3. Left Panel: The Optimal Penalty Function $\tilde{f}(i)$ for the Supervisor for Various Values for the Upper Bound $\alpha^*$ Using the Asymmetric Loss Function with $\alpha_1 = 4$. For ease of reference, the penalty function proposed by the BCBS (1996a) is also presented. $i$ denotes the number of VaR violations during the last $n = 250$ trading days. The planning period used for the figure is $T = 10$, while discounting takes place at a rate of 10 percent.

Right Panel: The Interval of Optimal Underreporting $c$ over the Considered Grid of Values for the Sharpe Ratio $SR$ and the Degrees of Freedom Parameter $\nu$. The horizontal axis in the right panel gives the value of the upper bound $\alpha^*$ minus 3. For each value of $\alpha^*$, we give the result for three values of the asymmetry parameter $\alpha_1$. $B$ again denotes the penalty function according to the BCBS (1996a) report.

potential penalties, for example, $\alpha^* = 10,15$, an intermediate penalty is imposed if either five or six VaR violations are encountered, respectively. Third, the maximum increase in the scaling factor for the Basle proposals falls far below the optimal maximum penalty. In particular, the optimal maximum penalty appears to coincide with the upper bound $\alpha^*$ used in the computations.

The right-hand panel in Figure 3 gives insight into the maximum percentage mismatch between true and reported VaR. It appears that within the present framework, the Basle guidelines result in severe underestimates of the true VaR. Maximum penalties should be set substantially higher in order to drive the value of $c$ closer to zero. Note that the use of an asymmetric loss function indeed results in an asymmetric distribution of optimal degrees of underreporting for sufficiently large $\alpha^*$. If $\alpha_1 > 1$, overreporting occurs relatively more easily. This is brought about by a slight increase in the penalty function at five or six VaR violations, see also the comments on the left-hand plot in Figure 3.

The practical implementability of the step-type penalty function exhibited in Figure 3 warrants one cautionary remark. The derivations so far hinge on the assumption that the bank knows its true VaR, while the supervisor does not. Although it is reasonable to presuppose that the bank has a better understanding of its VaR than the supervisor, it is unrealistic to assume that the bank can estimate its VaR without error. If we relax the assumption of complete knowledge of the true VaR by the bank, a more gradual shape of the optimal penalty function might be more appropriate in order to account for the possibility of unintentional VaR misspecification by the bank. Alternatively, the supervisory institution could retain the step-type penalty function and put the whole burden of accounting for estimation and model mis-specification risk on the bank. This would stimulate the banks somewhat more to design models that produce more prudent VaR numbers, that is, lower $c$s.

## 4. CONCLUDING REMARKS

In this paper we have evaluated the Basle proposals for the use of internal models in conjunction with backtesting procedures (BCBS 1996a, b). It turns out that the present proposals for imposing penalties on banks that violate their VaR bounds too often are highly inadequate. The monetary penalties are too low to provide a sufficiently strong incentive for banks to design internal risk management models that produce good estimates of their true VaR. Consequently, it is profitable for banks to underreport their true market risk. The effect will be mitigated insofar as the market capital requirement lies above the regulatory capital requirement (cf. Estrella 1995). From a prudential perspective, however, it is good practice not to rely too much on the market capital requirement to be binding, especially if no solid figures on it can be obtained from empirical research. In any case, even if the market capital requirement is binding, the present paper shows that the current backtesting framework does not contribute to a proper alignment of the banks' incentives for VaR reporting with those of the supervisor.

The optimal strategy for the supervisor given the above findings is to set much higher monetary penalties on an excess number of VaR violations. Moreover, the penalty function can be made much steeper than in the BCBS (1996a) proposal. The steepness result, however, is based on the bank knowing its own true market risk. This is an unrealistic assumption for all practical purposes. If estimation and model misspecification risk are taken into account, more gradual penalty functions might turn out to be optimal. More research has to be directed to this topic.

As to the height of the maximum penalty, several remarks are in order pertaining to the practical implementability of the results. First, requiring a large penalty following a large number of VaR violations can be a draconian measure. As an alternative, one could use a smaller penalty, for example, in the form of a smaller increase in the VaR scaling factor that is left in place for a longer time. Similar ideas have been developed in the context of the precommitment approach (see Kupiec and O'Brien 1997). Second, the supervisor should always retain the right to abstain from imposing penalties if a bank can convincingly argue that the cause of a large number of internal model failures lies beyond its responsibility. The burden of proof, however, should remain at the bank (cf. BCBS 1996a). Finally, imposing penalties, in particular high ones, can be inappropriate in situations of severe market fluctuations. In particular, a large number of VaR violations may signal that the bank has gone through a difficult period. Imposing (severe) monetary penalties in addition to the already experienced difficulties might then push the bank further into trouble, and in the extreme into default. This comment, however, pertains equally to the backtesting as to the precommitment approach.

Several interesting questions for future research remain. For example, it is interesting to investigate the effect of active asset and liability management (ALM) from the side of the bank on the optimal supervisory policies. If banks are able through active ALM to limit the number of VaR violations if a certain number of VaR violations has already occurred during the supervisory period, then we expect even more severe underestimates of the true VaR for reporting purposes. Modeling the internal ALM process of the bank, however, is far from trivial, and more research is needed to design an adequate and tractable framework (see, for example, Fusai and Luciano 1998). It would also be interesting to add uncertainty to the model in the form of an unknown value for the true VaR. Although this would highly complicate matters within the present framework, it seems more realistic that neither the bank itself nor the supervisor can come up with a faultless estimate of the true VaR.

## APPENDIX: REWRITING THE OBJECTIVE FUNCTION

For simplicity, we set $r \cdot VaR^* = 1$. This does not affect the optimum value of underreporting $c$. We now have

$$E_0\{U[(1 - c(t)) \cdot f(t)]\} = E_0\{U[(1 - c(t)) \cdot f(t)] \mid f(s) < 4 \ \forall \ s \in \{1,...,t - 1\}\} \cdot P[f(s) < 4 \ \forall \ s \in \{1,...,t - 1\}] + E_0\{U[(1 - c(t)) \cdot f(t)] \mid \exists \ s \in \{1,...,t - 1\}: f(s) \geq 4\} \cdot P[\exists \ s \in \{1,...,t - 1\}: f(s) \geq 4 ] . \quad \text{(A1)}$$

Using (5), define $\xi_1$ as

$$\xi_1 = \sum_{i=0}^{9} \binom{n}{i} p(c)^i [1 - p(c)]^{n-i} , \quad \text{(A2)}$$

that is, the probability of not entering the red zone given that the original VaR reporting level is still used. We then have

$$\begin{aligned} \xi_t &= P[f(s) < 4 \ \forall \ s \in \{1,...,t\}] \\ &= P[f(t) < 4 | f(s) < 4 \ \forall \ s \in \{1,...,t - 1\}] \cdot P[f(s) < 4 \ \forall \ s \in \{1,...,t - 1\}] \\ &= \xi_1 \cdot \xi_{t-1} , \end{aligned} \quad \text{(A3)}$$

such that $\xi_t = \xi_1^t$. Equation (A1) can now be rewritten as

$$\xi_1^{t-1} \cdot U_0 + (1 - \xi_1^{t-1}) \cdot U^* = \xi_1^{t-1} \cdot (U_0 - U^*) + U^* , \quad \text{(A4)}$$

with

$$U_0 = \sum_{i=0}^{9} \binom{n}{i} \left\{ U[(1-c) \cdot \tilde{f}(i)] \cdot p(c)^i [1 - p(c)]^{n-i} \right\} + \sum_{i=10}^{n} \binom{n}{i} \left\{ U[(1-c^*) \cdot \tilde{f}(i)] \cdot p(c)^i [1 - p(c)]^{n-i} \right\}, \quad \text{(A5)}$$

and

$$U^* = \sum_{i=0}^{n} \binom{n}{i} U[(1-c^*) \cdot \tilde{f}(i)] \cdot p(c^*)^i [1 - p(c^*)]^{n-i} , \quad \text{(A6)}$$

where $n = 250$, and $\tilde{f}(i)$ takes the values given in Table 1 for $i = 0,\ldots,250$. Note that $U_0$ and $U^*$ are the expected (powers of) opportunity costs given the red zone has not and has been entered before, respectively. The objective function (4) can now be rewritten as

$$\min_c \left\{ U[f(0)\cdot(1-c)] + \sum_{t=1}^{T-1} e^{-\rho t}\cdot[\xi_1^{t-1}\cdot(U_0 - U^*) + U^*] \right\}, \tag{A7}$$

such that (6) follows by straightforward algebraic manipulations.

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